

1 Cambridge Number Theory Study Group - Michaelmas 2025

Description. Understand some of the basic theory and ideas involved in unlikely intersections on Shimura varieties. Brief overview: [Tsi24], detailed overview: [UY18]¹.

Talk 1: **Introduction:** background, motivation and overview via toral case, and o-minimality.

Venue: 16/10 14:00-15:00 in MR9

Speaker: Suvir Rathore

Talk 2: **Shimura Varieties:** background and examples of modular curve, A_g , special points/subvarieties and examples (§2.2-2.4 in [UY18]). *Ambitious:* application of unlikely intersections - state and sketch proof of Theorem 7.1 in [PT14] (can limit to case of special points) as a corollary to large Galois orbits.

Overview: §3 in [Tsi24], §2 pg 5-6 in [Pil23].

Venue: 23/10 16:00-17:00 in MR9

Speaker: Jack Thorne

Talk 3: **Heights on Shimura Varieties:** Falting's height following §9.2 (not including Theorem 9.2) in [UY18] and relation to canonical height on CM points following §1.2, 1.3 and §9 (without proofs) in [Pil+24].

Overview: §7 in [Tsi24], §1.2, 1.3 in [Pil+24].

Venue: 30/10 14:00-15:00 in MR14

Speaker: Dmitri Whitmore

Talk 4: **Functional Transcendence:** Basics of o-minimality following §3.1 in [Pil23] and sketch proof of Pila-Wilkie (after §3.3). Statement of classical Schanuel conjecture and (sketch proof of) classical Ax-Schanuel following [Tsi15], if time permits statement for A_g as in §6.2 [Tsi24]. *Ambitious:* instead give proof of Ax-Lindemann-Weierstrass following §7 [Orr15], or proof of Ax-Lindemann for A_g following §6 [PT14].

Overview: §6 in [Tsi24], §4.4 in [UY18].

Venue: 06/11 14:00-15:00 in MR9

Speaker: Martí Riera

¹Working link: <https://ullmo.ihes.fr/publications/SurveyKUY.pdf>

Talk 5: **Large Galois Orbits:** Review of proof of Theorem 1.1 in [Tsi18] (LGO for A_g) following §9 in [UY18] (skipping details of Falting’s height from talk 3). *Ambitious:* Can prove $g = 1$ case directly (see §2.1.4 [Tsi23]).

Overview: §7 in [Tsi24], §3.5 in [Tsi23].

Venue: 13/11 14:00-15:00 in MR14

Speaker: Bence Hevesi

Talk 6: **André-Oort for $Y(1)^2$:** Review André’s original proof for $Y(1)^2$ following §4.2 in [ZM12]². *Ambitious:* Effective proof following [Küh12].

Overview: 2.3.3 in [Tsi23].

Venue: 20/11 14:00-15:00 in MR9

Speaker: Jef Laga

Talk 7: **Pila-Zannier Strategy:** Review proof of André-Oort for A_g stating ingredients (without proof) in generality following §6 (without proof) and §8 in [UY18]. *Ambitious:* Review state-of-the-art of ingredients of Zilber-Pink conjecture following §3.3 in [Daw25]. Review generalisation to mixed Hodge structures following §1 in [Kli17]. Other generalisations (e.g Zilber-Pink for $Y(1)^3$ [DOP25]).

Overview: §4 in [Tsi24], §2.5 in [UY18].

Venue: 27/11 14:00-15:00 in MR14

Speaker: Lucas Valle-Thiele

Talk 8: **Finite Characteristic.** Formulations of André-Oort for $\mathbb{A}_{\mathbb{F}_p}$ following [ER20] and André-Pink-Zannier in characteristic p following [LS25]. *Ambitious:* Sketch proofs (e.g. Theorem 4.1 [ER20]) Abelian varieties arising as Jacobians following §1.1 [LS25].

Venue: 04/12 14:00-15:00 in MR9

Speaker: Rong Zhou

References

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²Available on iDiscover (online version and in Moore Library).

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- [Tsi18] Jacob Tsimerman. “The André-Oort conjecture for g ”. In: *Annals of Mathematics* 187.2 (2018), pp. 379–390. DOI: 10.4007/annals.2018.187.2.2. URL: <https://doi.org/10.4007/annals.2018.187.2.2>.
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- [Tsi24] Jacob Tsimerman. “The André-Oort Conjecture”. In: *Notices of the American Mathematical Society* 71 (Nov. 2024), p. 1. DOI: 10.1090/noti3052. URL: <https://www.ams.org/journals/notices/202410/rnoti-p1307.pdf>.
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